

Habit formation, information exchange and the social geography of demand

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August, 2008

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Habit Formation, Information Exchange and the Social Geography of Demand*

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August 21, 2008

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Introduction

Modern microeconomics consists of two major parts: firm behaviour and consumer behaviour. Although they emerged at roughly the same time in history our models (and understanding) of firm behavior tends to be much more sophisticated than those of consumer behaviour. One difficulty in modelling consumer behaviour lies on the observation that consumption is in many ways a social activity. This has been observed both in the context of bandwagon behaviour or conspicuous consumption (Liebenstein, 1950; Smith, 1776 or Veblen, 1899) but also in the context of learning to consume (Witt, 2001). Consumers often face incomplete information both about what is available, and how to “get the most out of” the goods they consume. Agents learn about what is available and how to use it from their friends and neighbours, among other sources. In addition, consumers appear to form habits, depending on rules of thumb and past behaviour to guide their future choices. These sort of interaction and path dependent effects make analysis of consumer behaviour particularly challenging.

In this paper we model the dynamics of individual consumer behaviour and look at its implications for the distribution of demand of goods over the social space. There are empirical studies of this issue, reporting on the impact of social space on demand (e.g. Birke and Swann, 2006), but those papers tend to explain their results through network externalities. In this paper we use more general constructs and show that network structure of social interactions can be reflected on demand. Key to the consumers decision-making and thus to the dynamics of demand, is the consumers on-going, or repeated valuation of her alternatives. In our model valuations have two parts: in the first, choice dynamics are driven by the influence of consumer’s own consumption history. In the other, dynamics are driven by the influence of neighbours’ consumption history. In short, our consumers routinely decide which products to buy. Consumption of a chosen product increases the future valuation of this product for a consumer. Consumers also routinely interact with their neighbours and exchange information about all the products on the market. Based on these information streams consumers further update their valuations for each product.

The model can be interpreted in two ways. One is to say that there is an imperfect informational structure in the economy and consumers are aware of that fact. In this case consumers try to reduce uncertainty (Jacoby et al., 1994) in their decision process by using two sources of information. One source is the information they receive through own experience (part 1). As consumers have the better understanding of the value of the goods they have already consumed, consuming the same good avoids possible disappointment. The other is the information they receive from their social networks about the available goods (part 2). Information gathered from “friends” can similarly reduce the risk of disappointment.

Another interpretation of the two parts of consumption dynamics would be that people form habits for the goods they consume (part 1) and that there is an interdependence in the utilities of nearby consumers (part 2). With regard to habit formation, we assume that in the consumption process a consumer forms some special skills for using the product and as a result receives somewhat higher utility every subsequent period she consumes the same product. Interdependencies discussed on this interpretation are again only positive and local. Here people get somewhat higher utility if their consumption bundle is similar

to their neighbours' consumption bundles. This is similar to the effect of a "peer group" identified by Cowan et al. (1997).¹

We solve this model for a particular set of initial conditions showing how consumption choices can cluster in social space, and further how the stability of that clustering pattern depends on parameter values. Solution of the model for arbitrary initial conditions is not possible in general, so we address this issue numerically, showing that the results and intuitions of the analytic solution carry over to the more general case.

The remainder of the paper is organized as follows. The first section briefly reviews the related strands of literature. The second section presents the model. In the third section we present an analytical solution to a special case of the model and some numerical explorations into the behaviour of the general case. The last section of the paper concludes.

1 Related literature

The early theoretical relaxations of the perfect informational structure assumption were applied to market organization (see Rothschild, 1973 for a survey), credit rationing (e.g. Jaffee and Russel, 1976, Stiglitz and Weiss, 1981) as well as to a general consumer behaviour (e.g. Nelson, 1970). Recently, consumers' need for and lack of different types of information have been studied more closely. For example, uncertainty about prices is discussed by Galeotti (2004), who looks at the welfare implications of search costs when the distribution of prices is unknown. Samuelson (2004) studies the implications of the lack of information about the consumption of the population one lives in, by modeling the interdependence among utilities of consumers. Similarly to Samuelson, in our model consumers do not have information about the consumption of the whole population: each agent observes only a small part of it.

As we noted earlier, consumers base their decisions (partly) on information coming from external sources. Research in marketing and psychology stresses the immense importance of information collection for the consumer decision process (Bettman, 1971), and there is a vast body of empirical literature on how to collect and use consumer information in marketing and psychology (see Babutsidze, 2007 for a review). People tend to collect information through many different sources, such as the media, sellers or other consumers. In this literature it seems that, if one accounts for the reliability of the external information source and its intention to influence the information receiver, the information received from peers, through social networks, is the most reliable (Hansen, 1972). Thus, if one wants to understand the influence of external information on consumer decisions, it seems reasonable to concentrate on information coming from peers, rather than from any other external source. While not denying the importance of other sources of more general external information, in this paper we focus on the effects of socially localized peer effects.

The two-part informational paradigm (internal and external) taken by this paper is not new in economics and has been applied to other related fields where agents have to make choices. For example, information cascade models (e.g. Banerjee, 1992, and Bikhchandani

¹Throughout the paper we combine these two sets of interpretations and call the the inertia force "habit formation" (the second interpretation) and interdependency force - "information exchange" (the first interpretation). The same applies to the title of the paper.

et al., 1992) consider two sources of information based on which a certain decision is made (in a setting of sequential choices among the consumers): private information and external signals. In principle, the model presented in this paper is also an information cascade model, but it differs from the conventional models in three ways. Firstly, choices that consumers make in our model are repeated. This allows us to study the effects of the change in internal information driven by the consumption process itself. Secondly, cascade models typically present agents with only two options whereas in our model agents can face an arbitrary (finite) number of options. Information about any option can form a cascade as information about it flows within the population, but this does not dissolve into a single binary choice for the agent. Rather agents use information (which may or may not be cascading) about each of the options to make a choice for one of them. Finally, in our model information about the decisions of other consumers is localized: consumers can not see the decisions made by everyone in the population, but rather only those of their neighbours.

Under the second interpretation of our mode presented above, the model relates to the literature about the habit formation. Habit formation in consumption was discussed early on by Deusenberry (1949) and Brown (1952). But these approaches were concerned with the formation of the general habit of consuming, meaning that people form habits to consume in general, not for some particular goods. More recently, the idea has been rigorously incorporated into consumer decision models by Abel (1990) and Lettau and Uhlig (2000). These studies are concerned with the formation of social consumption habits (i.e. habits shared by the society). By contrast the present paper is concerned with individual habit formation for a single-good. These are the habits that people develop themselves through the consumption process, such as eating habits. Smith (2002), for example, shows that people acquire very strong eating habits that persist for a long period. He also shows that people are more likely to consume products that they see other people consuming, which is a basic assumption of our model.

One more relevant strand of literature is about non-market interactions and the debate about the different effects of global versus and interactions. Here the literature contains models about non-market interactions between consumers and producers (e.g. Scheinkman and Woodford, 1994, Weisbuch, 2006) as well as interactions among the consumers (e.g. Eshel, et al. 1996, Cowan et al. 1997). In general, interactions generate some kind of feedback loops that affect the decisions of the economic agents. As noted by Glaeser and Scheinkman (2000) the structure of those interactions does matter for the outcome we obtain at the end. In particular, they show that when interactions are local the economy generates more interesting dynamics, having multiple equilibria and the possibility of moving from one equilibrium to another. More contextualized works on interactions show that they can explain certain interesting phenomena in economics or other social sciences, such as the standardization (or technology adoption) process (e.g. Arthur, 1989, Cowan, 1991), waves in consumption across the population classes (Cowan et al. 2004), “contagious justice”(Alexander and Skyrms, 1999) or standardized traits (Altruists vs. Egoists) (Eshel et al. 1998).

Methodologically, the present paper closely relates to Dorofeenko and Shorish (2005), who present a general framework using partial differential equations to study fixed strategy games. But the work presented here is distinct from Dorofeenko and Shorish (2005) in two major ways. Firstly, our model presented here can not be formulated as a fixed strategy

game, indeed, part of our goal is to observe conditions under which strategies do or do not change. Secondly, while they utilize the Fokker-Planck approximation to change the state space from discrete to continuous, the state space in our model is continuous from the beginning and the partial differential equation arises from Taylor series approximations.

2 The model

Consider an economy inhabited by a large, finite number (S) of agents. Each of them is a single consumer, who, at every time period, chooses which goods to consume out of a fixed set of available goods. All goods on the market are substitutes. They are indivisible and a consumer can only consume a single unit of each.

An important notion in this paper is that of “valuation.” The valuation a consumer ascribes to a given good is the maximum price the consumer is willing to pay for it. Using very basic consumer theory, the utility a consumer derives from the consumption of a good will be the difference between its valuation and price that she pays for the good.

We adopt a standard discrete choice approach (Andersent et. al., 1992) and assume that each consumer buys one and only one product each time period. Under this assumption the utility of individual agent can be written as

$$U_t^s = v_{n^*,t}^s, \quad (1)$$

where $v_{n,t}^s$ is the valuation of good n in period t for consumer s (net of price that she has paid for the good) and n^* is the good that consumer s has consumed in period t . Under this setup the maximization of the utility implies that the good that consumer s chooses in period t is $n_t^{s*} = \arg \max(v_{n,t}^s)$.

What we seek to model here is the dynamics of product purchases based on the changes in the valuations of all the consumers about all the goods available on the market. Following the discussion in the introduction, we assume that valuation is derived from information of two types: internal and external. So, we can write:

$$v_{n,t}^s = f(x_{n,t}^s, y_{n,t}^s), \quad (2)$$

where the value of $x_{n,t}^s$ is determined by own consumption history (part 1) and of $y_{n,t}^s$ by the consumption history of other people in the same social group as the given consumer (part 2).

Both parts of the valuation are subject to change over time: $x_{n,t}$ is subject to change due to habit formation and $y_{n,t}$ is subject to change due to information exchange.² Thus, if we assume that $f(\cdot)$ is additive, we can write the dynamics of v_n^s as³

$$\frac{dv_n^s}{dt} = \frac{dx_n^s}{dt} + \frac{dy_n^s}{dt}. \quad (3)$$

To model information exchange among consumers we assume that every consumer has a fixed social location and a fixed neighbourhood. A neighbourhood is the set (\mathbf{H}^s) of

² $y_{n,t}$ can be also interpreted as a network effect, and then its dynamics will be dependent on the preferences of the (part of) society.

³From here on we drop the time subscript, but it should be borne in mind that the model is inherently dynamic and it is implicitly present in all the variables used throughout the paper.

other agents with whom an agent interacts and exchanges the information directly. Each information exchange consists of two agents revealing to each other their private evaluations of each of the goods. The information revealed is assumed to be “convincing” in the sense that the post-exchange evaluations of each of the two agents partially converge. Hence, this exchange process can be expressed simply in terms of the dynamics of beliefs of a single agent, s , following her exchanges with all of her neighbours, i :

$$\frac{dy_n^s}{dt} = \sum_{i \in \mathbf{H}^s} \mu(v_n^i - v_n^s), \quad (4)$$

where \mathbf{H}^s is the set of agents in the neighbourhood of s , and $\mu (\in [0, 1])$ is the information exchange parameter. Note that because we assume that all the products on the market are substitutes and there are no *ex ante* systematic differences among consumers, the information exchange parameter (μ) is the same across all the goods and agents.

Next, we give a shape to the social space. Assume that all consumers are aligned on a circle such that the distance between any two agents corresponds to the social distance between them, and the distance between immediate neighbours is constant across all the population.⁴ In this case it is very easy to define the neighbourhood of an agent (\mathbf{H}^s) simply by specifying the number of agents with whom this consumer interacts on the left and on the right.⁵

If we assume the neighbourhood size to be equal across the population, we can write

$$\frac{dy_n^s}{dt} = \mu \sum_{h=1}^H [(v_n^{s+h} - v_n^s) + (v_n^{s-h} - v_n^s)], \quad (5)$$

where s can be interpreted as a “serial number” of an agent, or her address (consequently, $s+1$ and $s-1$ are her immediate neighbours from the right and from the left respectively).

Re-arranging, (5) can be rewritten as

$$\frac{dy_n^s}{dt} = \mu \left[\sum_{h=1}^H (v_n^{s+h} + v_n^{s-h}) - 2Hv_n^s \right]. \quad (6)$$

Valuations are also influenced by habit formation.⁶ Habits are formed only for goods that are consumed. Thus, dx_n^s/dt is equal to zero for the goods that are not consumed in a certain period and is equal to some positive value for the good that has been consumed:

$$\frac{dx_n^s}{dt} = \begin{cases} \zeta & \text{if } n = n^* \\ 0 & \text{otherwise,} \end{cases} \quad (7)$$

where $\zeta (> 0)$ is a constant.

⁴Note that the results of the model can be applied to any type of space besides the social (e.g. physical space). It only depends on the interpretation of the circle on which the consumers are located.

⁵In general it is not crucial to assume that people have to be close socially in order to interact (although this assumption is not very far from reality). The effects demonstrated in this paper would hold for any type of network, be it social, geographical or any other type. The only requirement is that the society has some kind of fixed structure over time and that every agent has the same number of connections.

⁶We should make clear, that by habit formation we mean individual habit formation, rather than social habit formation which is a common assumption in ‘catching up with Joneses’-type of models (e.g. Abel, 1990, Fuhrer, 2000).

To summarize the model specified to this point we can make explicit sequence of regular routines of consumers. At the start of each period every agent decides which good to consume. After purchase she consumes it and forms habits for it. Towards the end of the period each agent meets all of her neighbours and passes to them all the information that she possesses. Based on the information communicated to them by neighbours all agents adjust their valuations of all goods.

We are interested in whether this kind of behaviour has implications for the social geography of demand; more precisely, whether any specific patterns emerge in the long-run. Essentially we ask whether one can tell anything about the consumption basket of a consumer by looking at the consumption baskets of her neighbours.

3 Analysis of the model

In this section we analyse the equilibrium and transition properties of the model. It is not possible to solve a completely general form of the model, so in the process of solution we do three things. First, we assume that the habit formation process can be well-approximated (at least in the region of interest) by a linear function. Second, we re-write the model as continuous in time and space. Finally we assume a specific initial condition.⁷ Following this solution to specific cases, we provide numerical results on the more general case.

Habit formation. Above, equation (7) shows habit formation: a consumer forms habits only for the good he consumes, and the effect on her valuation takes place in discrete jumps. This describes a path dependent process. However, if we employ a strategy used in the discrete choice literature (Andersen et al., 1992) to model expected habit formation rather than realized habit formation we can approximate the dynamics of (7) with a Markov process. We model the choice of the consumers as a conventional discrete choice, where the choice is based on probabilities : agent s chooses good n with probability $p_{n;t}^s$ at time period t . In this case, the law of motion in equation (7) becomes:

$$\frac{dx_n^s}{dt} = \begin{cases} \zeta & \text{with probability } p_{n;t}^s \\ 0 & \text{with probability } 1 - p_{n;t}^s \end{cases} \quad (8)$$

Further, $p_{n;t}^s$ will be a function of the vector of valuations for the agent s at period t . Thus we can write $p_{n;t}^s = p_n(V_t^s)$, where V_t^s is the vector of valuations. Then the expected change of $x_{n;t}^s$ can be written as:

$$E\left(\frac{dx_n^s}{dt}\right) = \zeta p_n(V_t^s). \quad (9)$$

Of course, the choice probability for a product n depends on valuations of all the products. But, the contribution of valuations of other goods (except that of n) are marginal, especially if there are many products available on the market. Thus, our probability function can be approximated by $p_n(V_t^s) \approx g(v_{n;t}^s)$. To obtain an analytic solution, it is

⁷This strategy for analysis of the model and its solution is borrowed from the model of chemical morphogenesis (production and diffusion of chemicals among cells) from biology. In particular we rely on Turing (1952) and Childress (2005).

necessary to linearize $g(v_{n;t}^s)$. The linearized form is simply $g(v_{n;t}^s) \approx \gamma v_{n;t}^s$, where $\gamma > 0$. Then, the expected change of $x_{n;t}^s$ can be written as

$$\frac{dx_n^s}{dt} = \alpha v_n^s, \quad (10)$$

where $\alpha = \gamma\zeta$ and can be interpreted as the rate of habit formation.⁸

Taking into account equation (10), our system can be written as

$$\frac{dv_n^s}{dt} = \alpha v_n^s + \mu \left[\sum_{h=1}^H (v_n^{s+h} + v_n^{s-h}) - 2Hv_n^s \right]. \quad (11)$$

From (11) it is clear that the law of motion of valuation for every good for any agent depends on its own level and on the valuations of the agent's neighbours of that same good.

For the demonstration of the solution, assume that each agent has exactly two neighbours ($H = 1$), and that there are only two goods available on the market ($N = 2$). These two assumptions reduce the model a system of S pairs of equations of the form

$$\frac{dv_1^s}{dt} = \alpha v_1^s + \mu(v_1^{s+1} + v_1^{s-1} - 2v_1^s) \quad (12)$$

$$\frac{dv_2^s}{dt} = \alpha v_2^s + \mu(v_2^{s+1} + v_2^{s-1} - 2v_2^s), \quad (13)$$

where $s = 0, 1, 2, \dots, S-1$ and S is the number of consumers in the economy.

Continuous time and space. We seek to obtain the solution to this system given by (12) and (13). As the choices are probabilistic, and probabilities are proportional to utility levels, higher valuation for one good compared to the other (for a certain consumer) would mean a higher probability that it will be bought by this consumer. Thus, as we have only two products, we are only interested in the difference between the valuations of these products. This is convenient as we can define a new variable $z^s = v_1^s - v_2^s$ and rewrite the system (12)-(13) as

$$\frac{dz^s}{dt} = \alpha z^s + \mu(z^{s+1} + z^{s-1} - 2z^s). \quad (14)$$

To solve this relation for z , introduce the variable Δ , which is the distance between two neighbours on the circle. This will allow us to discuss s as a variable, which will be (yet another) argument of z . With this modification the equation above becomes

$$\frac{dz^s}{dt} = \alpha z^s + \mu(z^{s+\Delta} + z^{s-\Delta} - 2z^s). \quad (15)$$

Then we can make a second order Taylor approximation in space around s for the terms $z^{s+\Delta}$ and $z^{s-\Delta}$. This will result in

⁸From here on, we drop the expectation sign, although it should be remembered that all the discussion in this section is about the expected values of the variables.

$$z^{s+\Delta} \approx z^s + \Delta \frac{dz^s}{ds} + \frac{\Delta^2}{2} \frac{d^2 z^s}{ds^2} \quad (16)$$

and

$$z^{s-\Delta} \approx z^s - \Delta \frac{dz^s}{ds} + \frac{\Delta^2}{2} \frac{d^2 z^s}{ds^2}. \quad (17)$$

Substituting equations (16) and (17) into equation (15) and considering the case when $\Delta \rightarrow 0$, which is, when the number of agents becomes very large, we get a partial differential equation of a following form

$$\frac{\partial z}{\partial t} = \alpha z + \tilde{\mu} \frac{\partial^2 z}{\partial s^2}, \quad (18)$$

where $\tilde{\mu} = \mu \Delta^2$.⁹

The local existence and uniqueness, as well as the analyticity of the solution to any partial differential equation (such as (18)) is guaranteed by the Cauchy-Kowalewski theorem (Dorofeenko and Shorish, 2005). But this also requires analytic initial conditions.¹⁰ In the following section we discuss the solution of (18) when initial conditions are analytic. Some insights to the dynamics of the model when initial conditions are not analytic will be provided in section 3.2.

3.1 Analytical solution to a special case

To analyze this system it is useful to separate the dynamics of z_t^s into the dynamics of the average \bar{z}_t over the population and the dynamics of the deviations from this average $\tilde{z}_t^s = z_t^s - \bar{z}_t$.

Let's look at the dynamics of average (\bar{z}) first. By the definition $\bar{z} = (1/S) \sum_s z^s$. This implies that

$$\frac{d\bar{z}}{dt} = \frac{1}{S} \sum_s \frac{dz^s}{dt}.$$

Using equation (15) we can write

$$\frac{d\bar{z}}{dt} = \alpha \frac{1}{S} \sum_s z^s + \mu \frac{1}{S} \sum_s (z^{s+\Delta} + z^{s-\Delta} - 2z^s). \quad (19)$$

Noticing, that the second summand of the right hand side of the equation (19) is zero and using the definition of average once again permits us to write (19) as

$$\frac{d\bar{z}}{dt} = \alpha \bar{z}. \quad (20)$$

This is a simple ordinary differential equation that has a solution

$$\bar{z}_t = e^{\alpha t} \bar{z}_0, \quad (21)$$

⁹From now on we only use z but it should be kept in mind that it really is z_t^s .

¹⁰"Analytic initial conditions" simply means that the initial condition can be expressed as a function.

where \bar{z}_0 is the difference between averages (over the population) of the valuations of the two products at time zero. Thus the difference in average valuations of the products grows exponentially in time. Clearly if at any time \bar{z} is not zero, one of the products is somehow perceived as superior, and this perceived superiority increases, exponentially, without bound.

Now we turn to the dynamics of the deviation from the average (\tilde{z}). For obtaining the analytical solution for deviations from the averages we need to specify an analytic initial condition. As we are working with the deviations from the average, we know that initial conditions should satisfy the constraint that $\sum_s \tilde{z}_t^s = 0$, which means that the average of these values across the population should be zero all the time.

There are probably many interesting initial conditions worth analyzing, but one particularly appealing one for the study of clustering is

$$\tilde{z}_0^s = \cos\left(k \frac{2\pi}{l}(s - \hat{s}_0)\right) \hat{\tilde{z}}_0, \quad (22)$$

where

$$\hat{s}_0 = \arg \max_{x \in [0, \frac{l}{k}]} \cos\left(k \frac{2\pi}{l} x\right)$$

and

$$\hat{\tilde{z}}_0 = \tilde{z}_0^{\hat{s}_0}.$$

Also, π is a measure of angle in radians, l is the length of the circle on which our consumers are aligned and k is the frequency of the sine wave in the initial condition, which takes non-negative integer values. Equation (22) implies that there is an initial clustering in preferences.¹¹

It can be shown (Childress, 2005) that in this special case (when the initial condition is given by (22)) the solution to our system (18) is

$$\tilde{z} = e^{\sigma t} \cos\left(k \frac{2\pi}{l}(s - \hat{s}_0)\right) \hat{\tilde{z}}_0, \quad (23)$$

where σ is the parameter yet to be determined. To find it we can simply substitute the solution (23) into the equation (18) and notice that $\partial^2 \cos(\beta x) / \partial x^2 = -\beta^2 \cos(\beta x)$. This gives us the value of σ :

$$\sigma = \alpha - \tilde{\mu} k^2 \left(\frac{2\pi}{l}\right)^2. \quad (24)$$

In order to combine the solutions for averages and deviations from averages consider the case where $\bar{z}_0 \neq 0$. That is, one of the products is perceived as superior. By equation (20) we can see that this perceived superiority increases without bound. In this case, although initially some of the consumers prefer the “inferior” product and their choices are reinforced by habit formation (and maybe even by information received from neighbours), at some point in time they will certainly be pushed to switch their choice. To see this mathematically assume that $\bar{z}_0 > 0$ and consider the choice of the agent who has the

¹¹The exception to this is the case when $k = 0$, which means a zero frequency sine wave, or equivalently, that \tilde{z}_0^s is constant across consumers. Then, as the average of deviations should be zero, $k = 0$ also means that all the deviations are zero.

lowest $z = z_{min}$, which will be negative.¹² Note that $\bar{z} - z_{min}$ is equal to the half the amplitude of the wave. Also assume that $\sigma > 0$.¹³ From equation (23) we know that the amplitude of the waves around the average is increasing at rate σ , while the average itself is increasing with rate α (equation (20)). Equation (24) establishes that $\alpha > \sigma$ unless $\mu = 0$. $\alpha > \sigma$ guarantees that \bar{z} increases faster than the amplitude of the pattern wave, which implies that eventually z_t becomes positive even for this extreme consumer.

Thus, $\bar{z}_0 \neq 0$ is a relatively trivial case, and implies that ultimately only one product is consumed in the population, no matter the dynamics of the deviations from the average. Far more interesting is the case in which $\bar{z}_0 = 0$, and thus $\bar{z}_t = 0$, which permits both products to co-exist indefinitely. From here on this is the case that we analyze. If $\bar{z}_t = 0$, it is true that $\tilde{z}_t^s = z_t^s$. Thus, simple replacement of \tilde{z} by z in equation (23) will give the complete dynamics of the case which we are interested to analyze.

If we fix the distance between the agents to be equal to unity ($\Delta = 1$),¹⁴ this will make $\tilde{\mu} = \mu$ and $l = S$, where S is the number of agents in the economy. This will help to simplify the solution to (23). Using these modifications, the result obtained in (23) can be rewritten as

$$z_t^s = \exp \left(\left(\alpha - \mu k^2 \frac{4\pi^2}{S^2} \right) t \right) \cos \left(k \frac{2\pi}{S} (s - \hat{s}_0) \right) \hat{z}_0, \quad (25)$$

where

$$\hat{s}_0 = \arg \max_{x \in [0, \frac{S}{k}]} \cos \left(k \frac{2\pi}{S} x \right)$$

and

$$\hat{z}_0 = z_0^{\hat{s}_0}.$$

This is the final solution of the model. It determines the value of z for every agent for every period in time. As one can notice at every period the distribution of z along the circle has a form of waves, which points to the fact that in some neighbourhoods z is positive, while in some other neighbourhoods it is negative. That means that some neighbourhoods are more likely to buy one product, while some other neighbourhoods are more likely to buy the other with a gradual transition between them. And the size of these neighbourhoods is equal to $S/(2k)$.

According to the Cauchy-Kowalewski theorem, the solution given in (25) is locally unique. To ensure the global uniqueness of the solution we have to have boundary conditions in space (Dorofeenko and Shorish, 2005). Recall that we are studying the unidimensional torus (a circle) and in space agents' addresses run from zero to $S - 1$. Then, agent S should also be agent zero. Thus the intuitive boundary condition is $z_t^0 = z_t^S$. It is easy to see, that our solution (25) satisfies this boundary condition, thus is globally unique.

¹²If $z_{min} > 0$ everybody prefers one product over another, which is a stable pattern, thus not interesting to discuss.

¹³If $\sigma < 0$, from equation (23) amplitude of the wave goes to zero and the difference between valuations of two goods become homogeneous across the population.

¹⁴Note, that $\Delta = 1$ does not undermine the validity of the continuous approximation of the system where we assumed $\Delta \rightarrow 0$. $\Delta = 1$ in this case means going from actual measure of distance back to consumer location indexes for the sake of interpretation of the results.

3.1.1 Discussion of the solution

Stability. A key concern is whether any observed clustering is persistent over time. Whether or not this is the case depends on the stability of the solution of equation (18), namely equation (25), which is written as:

$$z_t^s = e^{\sigma t} \cos \left(k \frac{2\pi}{S} (s - \hat{s}_0) \right) \hat{z}_0.$$

This is clearly dependent on the sign of σ ; if $\sigma < 0$ the solution is stable, converging to $z_t^s = 0$ and, thus clustering in demand is a temporary phenomenon; whereas if $\sigma > 0$ we have an unstable solution to (18), the amplitude of the waves is ever-increasing and clustering in probabilistic purchases becomes more and more pronounced with time.

Recall that

$$\sigma = \alpha - \mu k^2 \frac{4\pi^2}{S^2}. \quad (26)$$

Thus, stability depends on the constellation of parameters α , μ and S , as well as on the value of k , which is part of initial condition.

A helpful construct to analyze the stability properties of the model is the cut-off value of k (let's denote it by \bar{k}). The cut-off is defined as the highest possible value of k under which the system is unstable, which means the initial clustering is permanent. As the stability of the clustering requires $\sigma \geq 0$ we can pin down \bar{k} by solving (26) for k , setting $\sigma = 0$ and recalling that k takes non-negative integer values. This gives us the cut-off

$$\bar{k} = \left\lfloor \frac{S}{2\pi} \sqrt{\frac{\alpha}{\mu}} \right\rfloor. \quad (27)$$

Equation (27) implies, that given the ratio between α and μ , and S the initial sine waves with frequency $0 < k \leq \bar{k}$ generate long-lasting clustering patterns, while any other value results in disappearance of neighbourhoods with time. The dependence of \bar{k} on α/μ and S is positive and monotonic. This implies that higher values of α/μ and/or higher values of S will support stability of smaller neighbourhoods.

As k is the frequency of the waves in valuation, clusters alternate regularly around the circle, with the size of each cluster being $S/(2k)$. If k is very large ($k = S/2$ in the extreme case) each individual constitutes one cluster. In this case each agent is susceptible to influence from both her neighbours, both proponents of the choice contrary to the one the agent has made. All the agents receive negative reinforcement from their neighbours. This is likely to produce an unstable pattern. By contrast, if k is small, most agents are in the middle of a cluster surrounded by like-minded agents. In that case most agents receive external information that reinforces their choice. This is likely to be a stable situation.

Intuitions. Besides k , σ depends on parameters α , μ and S . From equation (26) the relation with each of the parameters can be specified:

$$\frac{\partial \sigma}{\partial \alpha} > 0; \quad \frac{\partial \sigma}{\partial \mu} < 0; \quad \frac{\partial \sigma}{\partial S} > 0. \quad (28)$$

Therefore, higher α , lower μ and higher S would all contribute to σ being positive, and thus a hardening of the community structure over time, for a given k .

These results have an intuitively appealing interpretation. Recall that the parameter α governs habit formation. A higher α would further reinforce the existing geographic pattern. If habits form quickly, what an agent consumes today is likely to be what he or she consumes tomorrow as well. Information received from neighbours has a smaller (relative) effect. In the extreme, if α is large enough, the first good consumed by any agent will be the only good consumed by that agent. The spatial pattern of consumption is frozen from the first day. Any clustering that exists is stable. On the other hand if α is so small that σ becomes negative, the clustering with a given k is not stable. In this case, agents decisions are dominated by information they receive from neighbours. But over the entire population, information is contradictory — some prefer good one, some prefer good two. Gathering all the contradictory information, any agent soon comes to the opinion that the goods deserve roughly equal valuations. All choices are made by coin flipping, and any spatial pattern disappears. μ is the parameter for information diffusion. A higher μ would mean a faster diffusion of the information and, consequently, a faster homogenization of information structure of population. Again, by the same process as happens with a small α , this process would work against any geographical pattern. Increasing the population size, (S), is a further obstacle in the way of information diffusion, and so slows down the homogenization process. Essentially what we observe here is a tension between own and public information. When agents lean heavily on the former, it tends to freeze choices, and any spatial pattern that exists will be relatively stable. When agents lean heavily on the latter, the initial condition that on average the goods are perceived as equally valuable drives agents towards indifference between the goods, and this the dissipation of spatial patterns towards random choices.

Now, recall that we have solved the model only for $H = 1$ and $N = 2$. It is also interesting whether these two variables have any influence on the stability of the system. In the appendix we have given the solution to the system with arbitrary H and arbitrary N . The case of $N > 2$ does not change the stability properties of the system, as σ remains as defined in equation (26). Thus, an increase in the number of goods does not have any effect on stability.

When H takes on arbitrary values, the solution for σ becomes

$$\sigma = \alpha - \mu k^2 \frac{4\pi^2}{S^2} \sum_{h=1}^H h^2. \quad (29)$$

From here, it is obvious that $\partial\sigma/\partial H < 0$, thus a larger neighbourhood implies that σ is positive for a smaller region of $\alpha \times \mu \times S \times k$ space. This is also intuitive as a larger neighbourhood facilitates the information diffusion process, which works to homogenize the information structure across the population.

3.2 General case

In the previous section we have given the analytic solution to a special case of the model. This solution (25) was based on a specific analytic initial condition, namely that relative preferences for the two goods were distributed over the population in a sine wave. An obvious question is how the model behaves starting from other initial conditions. Addressing this question is part of the goal of the current section. The most general case is when the initial valuations are random. In this case initial conditions are non-analytic, thus an an-

analytical solution is not possible, but we can analyze the model numerically. This will also give us an opportunity to demonstrate the validity of the methodology used for solving the special case analytically and to ensure that the results and intuitions developed there carry over to more general models, and in particular to the model as originally described in section 2.

This section presents typical runs of three specifications of the model. The structure of these experiments is to move incrementally from the model as solved in section 3.1, towards the model as originally described in section 2. Thus we present first the expected development of the model of section 3.1 but with a particular specification of the choice probabilities. Next we show the actual development of the model when habit formation takes place only on the good that the consumer is consuming at any moment. Finally, we show a simulation of an original model.

The main question in each case is whether any clustering emerges, and if so, whether it is stable. In all three cases the answer to the first question is that clustering does emerge. But we must point out that it is not stable, but rather meta-stable. One of the conditions for stability (see section 3.1) is that the average valuation of the goods over the population must be equal. If they are not equal, then the good with the higher average valuation dominates all other goods (exponentially) over time (see equation (21)). With non-analytic initial conditions we can not guarantee that $\bar{z}_t = 0$ for all time periods (although it can be imposed for the first period ($\bar{z}_0 = 0$)). Thus, since choices are random (even though the probability distribution is governed by valuations) at some point one of the products will become perceived as “superior” on average and we collapse to the case identified as trivial in previous section.

Settings. In each of the experiments reported below, we use the following parameter values. We set the number of goods to $N = 10$.¹⁵ The population size is $S = 100$; the population is located on a one-dimensional periodic lattice, so the neighbours of agent 0 are agents 1 and 99. The specific parameters for habit formation, α and information diffusion μ are $\alpha = 0.001$ and $\mu = 0.01$. Finally, each agent has one neighbour on either side, $H = 1$.¹⁶ To read the figures below, agents are arrayed along the abscissa, remembering that the axis is a circle, so the right-most and left-most agent are neighbours. Time is read on the ordinate, from the initial period, $t = 0$ to the final period, $t = 2000$. Each good is assigned a different shade of gray. The ordering of the goods, and therefore the shades of gray, is arbitrary. At each point in time the choice (or the good with the highest valuation) for each agent is shown by the colour corresponding to that good.

Finally, we specify the function mapping valuation to the probability of choice. Here we simply adopt the multinomial logit, from discrete choice theory:

$$p_n(V_t^s) = \frac{e^{v_{n;t}^s}}{\sum_{i \in \mathbf{N}} e^{v_{i;t}^s}}, \quad (30)$$

where \mathbf{N} is the set of all available products.

¹⁵We expand the number of goods for reasons of generality. We have shown (in the appendix) that number of products does not affect the stability of the system.

¹⁶Note that for this constellation of the parameters $\bar{k} = 5$.

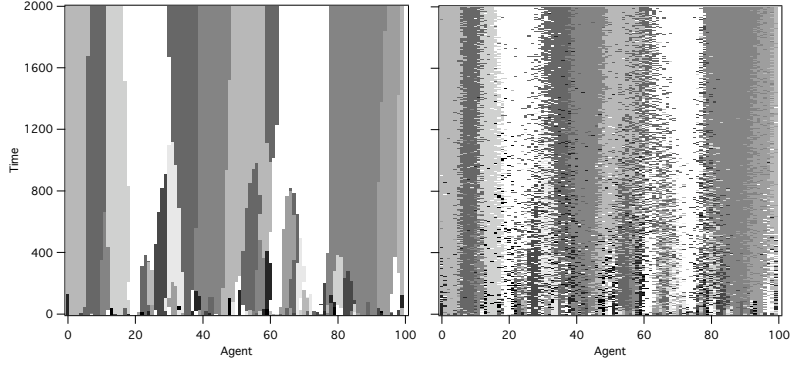


Figure 1: The most preferred goods (left) and actual purchases (right) with probabilistic purchases, and expected habit formation.

The equivalent model. In figure 1 we show the development of the model specified in section 3.1, with the laws of motion given by equation (11). The only difference between the numerical results presented here and the analytic solution is that that here we use random initial conditions: for each agent-product pair a $v_{n;0}^s$ is drawn from the uniform distribution over the interval $[0, 20]$.¹⁷ In the left panel of figure 1 we show the most preferred good; in the right panel we show actual choices. As one can see, despite the random initial values, the clustering pattern is clearly identifiable for the most preferred goods after few periods. The same pattern is replicated (although with some noise) by the actual purchases. Actual choices differ from the preferred good due to the probabilistic choice function (equation (30)). This difference is especially marked near the borders of a region, since here agents receive contradicting information about products, which tends to reduce the difference between their valuations of the most preferred good and other goods. This makes the probability choice function relatively flat for agents near the borders of clusters, and choices less correlated with those of their neighbours.

Specific habit formation. Figure 2 also shows the expected ddynamics of the model. But in this case we model habit formation as occurring only on the product currently

¹⁷Changing the uniform distribution to other standard symmetric distributions does not change the results of simulations.

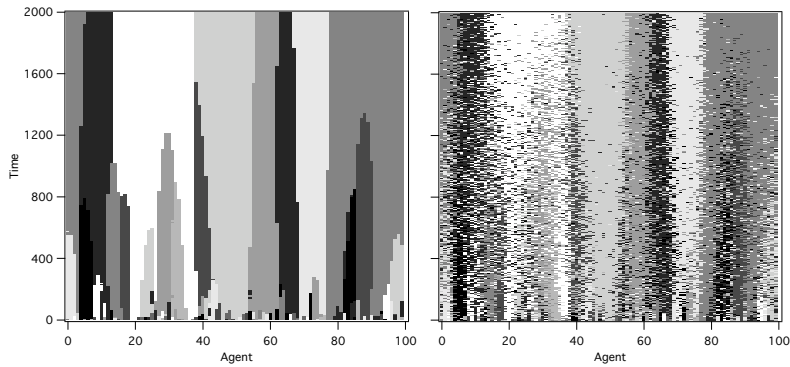


Figure 2: The most preferred goods (left) and actual purchases (right) with probabilistic purchases and habits formed only on the purchased good.

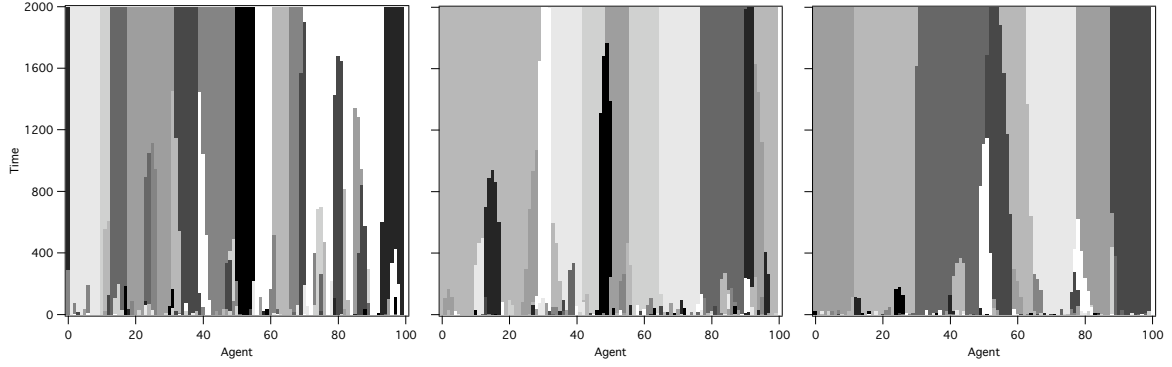


Figure 3: Consumers purchase their preferred good deterministically, with $\zeta/\mu = 1$ (left) $\zeta/\mu = 0.5$ (middle) and $\zeta/\mu = 0.25$ (right), showing the negative relationship between (minimum) neighbourhood size and ζ/μ .

being consumed, and in discrete jumps (as in equation 7), in contrast to the analytical model in which we modelled expected habit formation. This model is distinct from the previous, in that here we use a habit formation *step* of ζ instead of habit formation *rate* α . We know that $\zeta = \alpha/\gamma$ where γ is the constant coming from the linearization of the choice probabilities. Unfortunately there is no way to pin down the value of γ . Due to this we cannot make a judgment about the relation between magnitudes of α and ζ , thus the choice of the value of ζ is somehow arbitrary. We choose $\zeta = 0.005$ and use the same values for all other parameters as in the previous run. As one can see (in figure 2), again the clustering in purchases is clearly visible and relatively stable.

Original model. Finally, we simulate the original model, as developed in section 2 with the same parameter settings as in simulation 2. This final simulation should demonstrate the validity of the general approach of formulating a Markov process for the approximation of the behaviour of the original path-dependent model. In this case we move away from the probabilistic choice function to a deterministic one: the most preferred products are purchased (as implied by the consumer utility maximization problem) and habits are formed only for these goods (equation (7)). A typical run of the original model is presented on the middle panel of figure 3. Once again, a specific pattern in demand across neighbourhoods is clearly identifiable, even though we start, again, from random initial conditions.

Figure 3 also demonstrates that the discussion in section 3.1.1, about the effects of the parameters on outcomes in analytical model, carries over to this general model as originally described in section 2. As shown in equation (27) the stability properties of the model depend on the value of α/μ . We know that $\alpha = \gamma\zeta$ and as γ is constant throughout, these properties should depend on the value of ζ/μ .¹⁸ We take the middle panel of figure 3 as the benchmark (recall that the parameter values here are equal to those used for figure 2) and create two other cases for comparison. The left panel of figure 3 shows the case when ζ/μ is twice as large as the benchmark run, while the right panel shows the

¹⁸They will also depend on a size of the economy (S). However, the effect of S is simply that the minimum sustainable neighbourhood size is linear in S^{-1} , (equation (27)), so we limit ourselves to showing the effects of ζ/μ .

case when ζ/μ is half as large. As one can see larger ζ/μ permits smaller neighbourhoods to survive for a longer period. If one takes into account the fact that the presence of multiple clusters is a metastable phenomenon, these results can be thought as corollary to our discussion of results of analytical model in section 3.1.1, as there higher ζ/μ would result in smaller neighbourhoods being stable in the long run.

These numeric exercises also permit us to make a comment about what revealed preferences cannot reveal. Revealed preferences give us information only about the most preferred product, namely which it is, and completely neglect the story that is going on in the background. By this we refer to the fact that agents do have preferences over, and information about the goods they do not in fact consume. Without acknowledging the importance of those “unexpressed” preferences it is difficult to understand a sudden change in consumption which is not simply imitating neighbours. This is something that is possible in our approach, and in fact is observed in figure 3, in all three panels.¹⁹ We observe several cases of an agent adopting a new good which neither of her neighbours consumes. In addition, in many cases the agent himself has never consumed it in the past. The explanation lies in the fact that an agent close to the border of a region can receive contradictory signals. Consider the following simple example. Agent $s - 1$ ranks good 1 first and good 3 last; agent $s + 1$ ranks good 3 first and good 1 last. Both agents, though, rank good 2 second. It is clear that agent s , based on her external information, could easily rank good 2 before either 1 or 3. If the high rankings of good 2 by $s - 1$ and $s + 1$ have emerged (due to information received by their neighbours) at roughly the same time, agent s can then switch to good 2, regardless of what he was doing in the past. This explains the emergence and growth of such neighbourhoods in our framework. Thus, our model is consistent not only with shrinking and disappearance of smaller geographical neighbourhoods, but also with the emergence and growth of new ones.

Concluding remarks

As we have seen in this paper, information streams through fixed social networks do affect the social geography of demand. These external information sources, together with internal information processing structures such as habit formation, generate rich demand dynamics for markets containing goods that are close substitutes. In an environment of static budget constraints, information diffusion through fixed social networks generate clustering in demand: some neighbourhoods collectively prefer one good over another, while other neighbourhoods do exactly the opposite. Depending on the characteristics of the society, this pattern can be fragile, or robust. In short, what we have seen in this paper is that several parallel informational cascades can result in persistent spacial distributions where clearly identified neighbourhoods have higher concentrations of one particular type of information (information about one product). Or to put it differently, where the peaks of different positive informational cascades (Hirshleifer, 1993) are located

¹⁹In the left panel, product emergence of this sort is relatively common: agent 66 at period 10; agent 24 at period 100; agent 85 at period 5. In the middle panel, agent 92 at period 200; in the right panel, agent 66 at period 5. Close examination of the figures shows that this is in fact relatively common, though happening earlier in the right and middle panels than in the left panel. There is also a nice example in figure 1: agent 60, at about time 1000, changes her consumption from the good he has been consuming for the past 600 periods to a good he has never before consumed.

in different places in social space.

It worth noting that stable clustering phenomena can also be obtained with simpler models. For example one can model consumers as celular automata, who are basing their decisions on purely neighbours' current states (for example Miller and Cowan, 1998). Our model differs from these specifications in two ways: firstly, we can discuss the importance of communication intensity, which is impossible in celular automata and secondly, in our model consumers exchange information about the merits of (all) the products with their friends instead of just observing their consumption baskets.

Another interesting aspect worth pointing out is the localized nature of interactions. One interesting question is how introducing some global interactions would change in outcomes. This can be analyzed by looking at the behavior of the model as neighbourhoods become very large ($H \rightarrow S/2$). Increasing H puts a downward pressure on \bar{k} , and very likely pushes it below unity,²⁰ which means that no long-run clustering will be stable (in case of products having the same quality). Thus, in line with Glaeser and Scheinkman (2000), our model demonstrates that local interactions result in richer and more complex dynamics of the model compared with global interactions.

One important shortcoming of the model that has to be mentioned is that under this methodological framework we can not say anything about the selection of equilibria. We have identified the large set of possible scenarios with respect to the long-run development of the system, but we can not say which of them is more likely to occur. By looking at the assumptions that we had to impose in order to solve the model, one can argue that this will depend purely on initial conditions of the system, which is not surprising given the path-dependent features of the model.

Appendix

In appendix we give the analytical solution to the model with two neighbours and two goods assumptions being relaxed. We consider them one by one.

Appendix A: $H > 1$ case

Here we relax the two neighbours assumption. We consider a general case of every agent having $2H$ neighbours, thus we directly work with the equation (11) in the paper. After assuming that the distance between every two agents on the circle is equal to Δ , equation (11) can be rewritten as follows

$$\frac{dv_n^s}{dt} = \alpha v_n^s + \mu \left[\sum_{h=1}^H (v_n^{s+h\Delta} + v_n^{s-h\Delta}) - 2Hv_n^s \right]. \quad (31)$$

Let's consider only the second summand in the equation above (as it is the only part affected by the neighbourhood size). It has a form²¹

$$\mu \left(v^{s+H\Delta} + v^{s+(H-1)\Delta} + \dots + v^{s+\Delta} + v^{s-\Delta} + \dots + v^{s-H\Delta} - 2Hv^s \right). \quad (32)$$

²⁰For example, in the small economy that we have simulated ($S = 100$), $H = 49$ implies that the value of α has to be around 160 times higher than the value of μ in order the system to be stable for the largest possible neighbourhoods ($\bar{k} = 1$).

²¹Hereafter we drop the good subscripts as this applies to every good in the system.

Now, if we apply a Taylor approximation of all the summands (except the last one) around v^s many of the terms will cancel out. First notice that the number of terms approximated is $2H$ and each of the approximations will contain the term v^s . All of these terms together will cancel out with the last summand in the equation above. Plus, the first order terms of all the expansions will also cancel out as each of them will appear in a pair: one with positive and one with negative sign (as the neighbourhood is symmetric). So, basically we are left with the second order terms. If we go to a continuous space we will get

$$\mu \left(H 2 \Delta^2 \frac{\partial^2 v^s}{\partial s^2} + (H-1)^2 \Delta^2 \frac{\partial^2 v^s}{\partial s^2} + \dots + 1^2 \Delta^2 \frac{\partial^2 v^s}{\partial s^2} \right). \quad (33)$$

Collecting terms we get

$$\mu \left(\left(H^2 + (H-1)^2 + \dots + 1^2 \right) \Delta^2 \frac{\partial^2 v^s}{\partial s^2} \right). \quad (34)$$

Or

$$\mu \left(\Delta^2 \left(\sum_{h=1}^H h^2 \right) \frac{\partial^2 v^s}{\partial s^2} \right). \quad (35)$$

From this it stems that the only modification that this generalization makes can be captured by the definition of $\tilde{\mu}$ in the text being changed to

$$\tilde{\mu} = \mu \Delta^2 \left(\sum_{h=1}^H h^2 \right). \quad (36)$$

Although, notice here, that as we have used the Taylor approximation to derive this result, as we consider larger and larger neighbourhoods the accuracy of the result deteriorates.

Appendix B: $N > 2$ case

Here we relax the two goods assumption. Let's consider the N good case. After applying a Taylor approximation procedure to all the equations (in this case N) in specification (12)-(13), the model can be written as the system of N equations of a following form

$$\frac{\partial v_n^s}{\partial t} = \alpha v_n^s + \tilde{\mu} \frac{\partial^2 v_n^s}{\partial s^2}. \quad (37)$$

After defining two $N \times N$ dimensional diagonal matrices, one A with only α 's on the diagonal and the other \tilde{M} with $\tilde{\mu}$'s on the diagonal, and three vectors, V which is the vector of v_n^s ,²² $\partial V / \partial t$ and $\partial^2 V / \partial s^2$ which contain first derivatives with time and second derivatives with space, the system defined in (37) can be written in a matrix form

$$\frac{\partial V}{\partial t} = AV + \tilde{M} \frac{\partial^2 V}{\partial s^2}. \quad (38)$$

²²We drop the agent superscript here as it applies to every agent.

It can be shown (Childress, 2005), that with initial condition equivalent to (22) the solution to (38) is

$$V = e^{\sigma t + iw(s - \hat{s}_0)} V_0, \quad (39)$$

where V_0 is a vector of initial values and $w = k(2\pi/l)$. In writing the initial condition (39) we consider the case, when average valuations across consumers of all the products are equal to each other all the time, which is equivalent to the case $\bar{z}_t = 0$ considered in the main body of the paper. Note, that the real part of (39) can be written as

$$V = e^{\sigma t} \cos(w(s - \hat{s}_0)) V_0,$$

which is the same as (23).

For the analysis of the stability of the system we again need to determine σ . Doing the same trick as in the paper (taking the first derivative with time and the second derivative with space and plugging back to the original equation), we get the following expression

$$(A - B) V_0 = 0, \quad (40)$$

where A is the same matrix of coefficients, while B is a new diagonal matrix, which has $\tilde{\mu}w^2 + \sigma$ terms everywhere on the main diagonal. So we get a new $N \times N$ dimensional diagonal matrix of a form

$$\begin{pmatrix} \alpha - \tilde{\mu}w^2 - \sigma & 0 & \cdots & 0 \\ 0 & \alpha - \tilde{\mu}w^2 - \sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha - \tilde{\mu}w^2 - \sigma \end{pmatrix},$$

determinant of which has to vanish for the nontrivial solution of the system. The determinant of the matrix above is easy to calculate: the determinant of a diagonal matrix is the product of its diagonal entries, so

$$Det = (\alpha - \tilde{\mu}w^2 - \sigma)^N. \quad (41)$$

Equating the determinant to zero and plugging the definition of w gives the opportunity to solve for σ

$$\sigma = \alpha - \tilde{\mu}k^2 \left(\frac{2\pi}{l} \right)^2, \quad (42)$$

which is absolutely the same as the solution obtained for the $N = 2$ case. Thus, this system, of course, has N solutions but all of them are given by (42).

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